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Modeling the Transient Inelastic Flow of Rocksalt

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ABSTRACT

The rheological behavior of rocksalt, like that of other crystalline materials, is dependent on its mechanical history, so that hereditary effects have a marked influence on its transient inelastic flow. The steady state flow, on the other hand, is usually considered to be independent of the previous loading history, at least in the ductile regime where the inelastic flow is associated almost exclusively with intracrystalline dislocation motion.

In the traditional models used to describe the behavior of rocksalt, hereditary effects have often been neglected or considered indirectly by using empirical transient creep laws, most of the time expressed as functions of external variables such as time or as accumulated isotropic strain. However, it has been demonstrated that in many instances hereditary effects cannot be neglected, nor can they be expressed solely by these empirical formulations, if one wants to properly describe the rheological behavior of rocksalt. This is particularly true for hyperstatic structures, such as thick walled cylinders or underground excavations, where the non-linear inelastic behavior of rocksalt induces a stress redistribution which modifies progressively the stress state. This stress variation in the structure interacts in a complex manner with the hereditary transient behavior of the material. As the type of model previously mentioned fails to consider properly such a process in the structure, more sophisticated models have been proposed in recent years, such as differential models with evolutionary state variables.

In this paper, some of these various approaches to constitutive modeling of rocksalt are discussed in relation to inelastic flow around underground excavations. It is inferred that evolutionary state variable models are best suited for describing properly the behavior of rocksalt in such conditions. A recently developed model, proposed by the authors, is considered to be the most comprehensive tool presently available for that purpose.

INTRODUCTION

The use of elasticity for modeling the mechanical behavior of geological materials should be considered relevant only for stress states which are below their yield strength. In reality, most geological materials, such as rock, soil and ice show rather a non-linear behavior, their yield strength being usually much smaller than their ultimate strength. This non-linearity stems from crack initiation and propagation, which can lead to failure at a given stress or strain level, or from ductile flow of the material which causes generalized inelastic straining over the volume.

It has also been observed that time can have a major influence on the mechanical properties of many of these geomaterials. In that respect, some crystalline rocks are particularly sensitive to strain rate effects, especially some of the soft rocks that are the subject of inelastic deformation mechanisms within the crystals at low to intermediate temperature under moderate confining stress (Kirby and McCormick, 1984; Carter and Tsenn, 1987). Rocksalt (halite) and potash (sylvinite) might be taken as representative of these soft and rather ductile rocks.

The design and monitoring of underground openings require the knowledge of the stress field and also, in many instances, of the strain and displacement fields, that will be induced around them, whether the rock shows elastic or inelastic straining. In the latter case, the stress, the strain and the displacement fields (and their evolution in space and in time), as predicted through numerical modeling, directly depend on the considered constitutive model. Using adequate constitutive equations is thus of prime importance.

The purpose of this paper is to discuss the problems associated with the selection of an appropriate constitutive model for the prediction of stress, strain and displacement fields around underground openings excavated in rocksalt and potash, and to show that a model allowing the proper consideration of transient inelastic flow is needed.

THE BEHAVIOR OF ROCKSALT

The mechanical behavior of rocksalt has been the subject of many experimental studies, and although most of these studies have been directly motivated by underground storage projects and mining activities, the bulk of the experimental effort has been focused on laboratory testing, using the well known conventional triaxial compression (CTC) test. Results of such tests on rocksalt have shown a strong non-linearity of its mechanical behavior. This non-linearity has been revealed, for instance, by the stress–strain curves obtained from constant strain rate tests, by the strain–time curves obtained from creep tests, and also by the stress–strain rate relationship obtained from both of these test results for steady-state flow (see Fig. 1).

When rocksalt is in a virgin (or completely annealed) state, it shows a very low yield strength, as inelastic strain appears almost from the beginning of the deviatoric loading. However, as shown in Fig. 1a, which is typical of many experimental results reported in the literature, straining causes the yield strength to rise. One can then observe an important hereditary (or memory) effect, as the inelastic behavior of the material is strongly influenced by its mechanical history (Aubertin et al., 1991a; 1991b). This is also a common feature of many other crystalline materials, such as metals for instance (Lemaitre and Chaboche, 1988).

The non-linear behavior of the material induces a stress redistribution in any hyperstatic structure which interacts with memory effects. Underground excavations are such structures; when openings are created in a rocksalt deposit, the stress field generated during their driving will then change in time and in space according to the material behavior and to the prevailing geometric and boundary loading conditions. Although it is well recognized that describing correctly, through mathematical modeling, the rheological behavior of rocksalt in such complex loading conditions is not an easy task, an adequate constitutive model remains an essential prerequisite for the prediction of the stress and strain (or displacement) fields around underground openings.

MODELING THE INELASTIC BEHAVIOR OF ROCKSALT

As previously mentioned, the mechanical behavior of rocksalt is usually studied in the laboratory using triaxial compression tests under constant strain rate (CSR) or constant stress (creep) testing conditions. In the case of creep data, most often used to study the delayed inelastic behavior of rocksalt, it

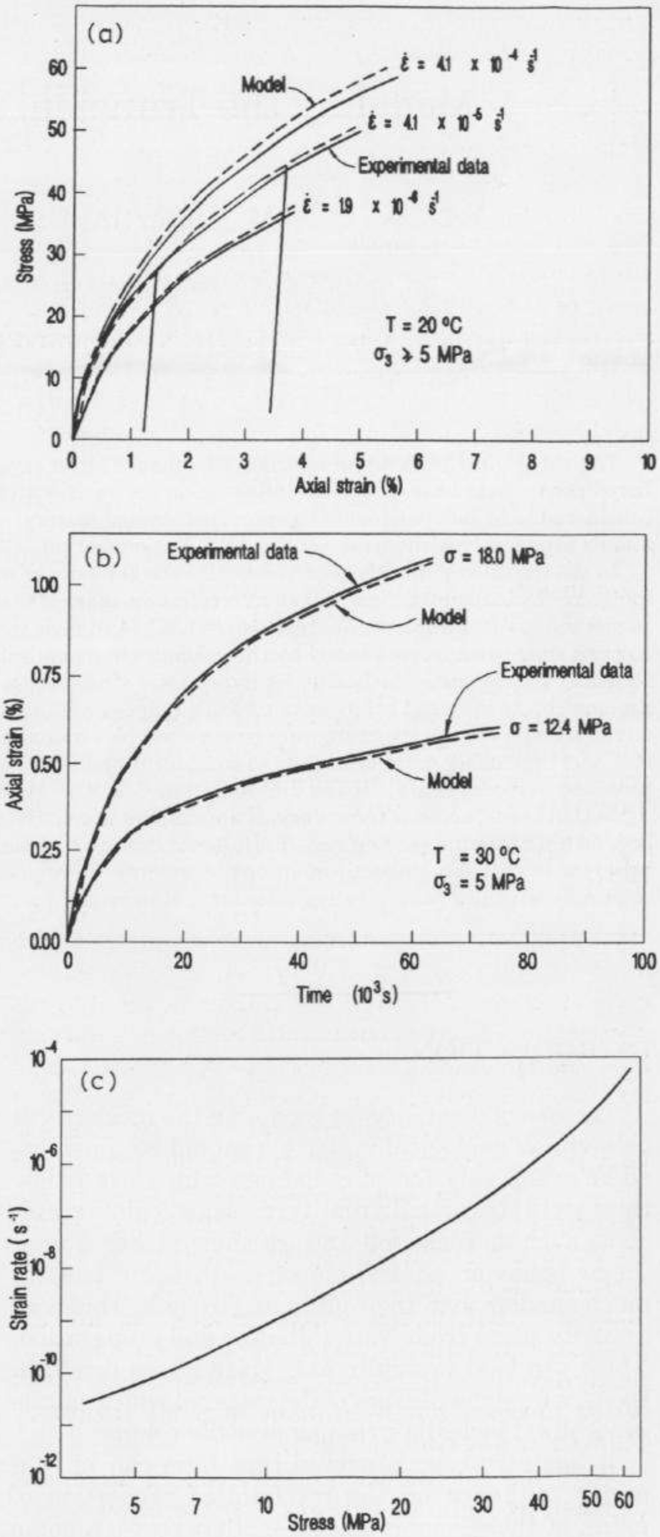

Fig. 1. Non-linear behavior of an artificial rocksalt under conventional triaxial compression (CTC) tests (data taken from Aubertin, 1989); dashed lines show the predictions from the SUVIC model. (a) Stress–strain curves for various strain rates. (b) Creep curves for various stress states. (c) Stress vs. strain rate relationship for steady-state flow.

is customary to identify four different straining phases on the strain-time curve obtained. The pseudo-instantaneous strain phase (including elastic, ε^e, and plastic, ε^p, strains), which occurs during the loading stage, is followed by three phases of time-dependent (creep, ε^c) strains: transient (or primary, ε_t), steady-state (or stationary, ε_s), and accelerating (or tertiary, ε_a), the last one being frequently omitted in usual applications. One can then write, using the traditional approach for constitutive modeling of the rheological behavior of rocks, a partitioned formulation (written here in an incremental form) where the total strain rate becomes

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p + \dot{\varepsilon}^c \quad (1)$$

$$\text{with } \dot{\varepsilon}^c = \dot{\varepsilon}_t + \dot{\varepsilon}_s + \dot{\varepsilon}_a \quad (2)$$

Each component in these equations is usually described by a distinct empirical function (or law). These functions are often formulated as curve-fitted expressions, directly related to specific experimental results. Examples include linear elasticity for $\dot{\varepsilon}^e$ and classical plasticity with a normality rule for $\dot{\varepsilon}^p$ (Desai and Varadarajan, 1987; Aubertin and Gill, 1992), time- or strain-hardening functions for $\dot{\varepsilon}_t$ (Lindner, 1983; Munson and Wawersik, 1991), and a power law function for $\dot{\varepsilon}_s$ (Carter and Hansen, 1983; Bérest, 1987).

Often aimed at projects such as hydrocarbons storage or radioactive waste isolation facilities, the analysis and design of underground excavations in rocksalt commonly require a long term stability investigation. Consequently, most research interests have been mainly focused, in the past, on steady-state flow, which is usually considered to be independent of the stress path and strain history, at least in the ductile regime where fracture plays a negligible role.

Since the transient phase of the rheological behavior has received much less attention from the scientific community than the steady-state phase (not only in the field of rock mechanics, one might add), it is still rather poorly understood and explained. Nevertheless, many efforts have been recently devoted to this aspect, and consequently interesting progress has been made (e.g. the review by Munson and Wawersik, 1991).

By using the traditional partitioning approach, depicted by equations (1) and (2), one implies that the inelastic strain ε^i can be divided explicitly into two distinct and independent components, namely instantaneous (ε^p) and delayed (ε^c) strains. However, theoretical and experimental evidence shows that interaction exists between these two components of inelastic strains, such as an effect of creep strain on the yield strength (Guessous et al., 1987), and an influence of loading rate on the subsequent creep rate in a creep test (Aubertin et al., 1991a). This is also consistent with observations made on other ductile crystalline materials, where plasticity and creep strains have been known to interact (Ohashi et al., 1986).

It can be further recalled that the empirical nonlinear formulations generally used for transient creep (see the recent presentations of Fuenkajorn and Daemen, 1988, and of Munson and Wawersik, 1991) have been shown to be inadequate, for rocksalt and for other crystalline materials, when the stress path is elaborate (Mukarami and Ohno, 1982; Handin et al., 1986). Time (t) or equivalent (isotropic) strain (ε_e) included in these classical functions of time- or strain-hardening are variables that can only be of a limited validity (Swearengen and Holbrook, 1985; Bérest, 1987), as neither can account directly for a complex mechanical history. Also, the separation between the transient and steady-state phase of inelastic flow, although mathematically useful, is physically unsound because the same physical processes govern both phases of inelastic behavior. Accordingly, similar stress and temperature dependence should be expected for $\dot{\varepsilon}_t$ and $\dot{\varepsilon}_s$ (Argon and Bhattacharya, 1987), which cast some doubts on models that contradict this widely accepted principle (such as the Hunsche/McVetty model for example; see Lux and Heusermann, 1983).

Facing such evidence, a unified approach to the inelastic behavior of crystalline materials has been developed. With this approach, usually closer to the physical processes that control the inelastic flow on the microscopic scale, a unique kinetic law can be formulated to describe the entire spectrum of inelastic strains. Here, the general expression for strain rate becomes (Bammann and Krieg, 1987; Delobelle, 1988):

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^i \quad (3)$$

In such a formulation, the plastic component of strain (or of strain increment) constitutes only a particular (and limiting) case, ensuing from the general expression that describes the inelastic strain ε^i (or strain rate). With equation (3), it is implicity admitted that the same mechanisms control the entire range of inelastic straining (that is ε^p and ε^c). This appears to be a valid assumption for crystalline materials in the ductile regime (Swearengen et al., 1985; Ohashi et al., 1986).

In the unified approach, the constitutive law of the material is usually composed of a kinetic law, for constant structure or state, and of at least one evolution law. The general kinetic law formulation can be expressed as follows (Aubertin et al., 1991b):

$$\dot{\varepsilon}^i = \dot{\varepsilon}^i[\sigma, T, Y] \qquad (4)$$

where σ represents the stress state, T the absolute temperature, and Y a set of state (or internal) variables, which are often (but not always) related to specific microstructural characteristics of the material. For each of the evolutionary state variables, one has to define an evolution law, that may be written as

$$\dot{Y} = \dot{Y}[\dot{\varepsilon}, T, Y] \qquad (5)$$

When formulated in a more specific manner, for isothermal conditions, this last equation often becomes

$$\dot{Y} = f_1[\dot{\varepsilon}^i] - f_2[Y, |\dot{\varepsilon}^i|] - f_3[Y] \qquad (6)$$

where f_1, f_3, and f_3 represent given mathematical functions associated respectively with the mechanisms of hardening, dynamic recovery and static recovery, thus following the well-known Bailey–Orowan principle.

Most of the state variable models presented in the literature have been developed for metals (see the reviews by: Bammann and Krieg, 1987; Delobelle, 1988). However, similar models have also been developed and/or investigated in order to describe the rheological behavior of rocksalt in the ductile regime (e.g. Senseny and Hansen, 1987; Aubertin, 1989; Russell et al., 1990).

In order to conclude this short review on the modeling of the inelastic behavior of rocksalt, one should mention that the various approaches used have been the subject of a number of classifications (e.g. the recent review of Munson and Wawersik, 1991). Considering the many similarities between the behavior of rocksalt and that of various metals (Poirier, 1985), the authors adopt here the classification defined by Swearengen and Holbrook (1985), who identify four broad classes of models, namely those based on:

I. the viscous stress method (such as linear viscoelastic models and classical non-linear creep laws),

II. functional constitutive relations using hereditary integrals (such as the endochronic theory of creep and viscoplasticity),

III. mixture theories which allows a scale-up of microstructural processes to the macroscopic behavior,

IV. state variable evolutionary models, in which the instantaneous value of the state variables are sufficient to define the mechanical history of the material.

This classification will be referred to in the following paragraphs. It will then become evident that the prediction of stress, strain and displacement fields around underground openings in rocksalt should make use of class IV models.

PREDICTING THE STRESS FIELD AROUND UNDERGROUND EXCAVATIONS

As previously mentioned, the design and monitoring of underground excavations in soft rocks, such as rocksalt and potash, requires the knowledge of the stress and displacement fields that will be induced around them. These fields and their evolution in time, as predicted through numerical modeling, are directly related to the constitutive model used.

At this point, one cannot expect a model to reproduce all the characteristics of the inelastic behavior of rocksalt. The objective of a constitutive model is to give a mathematical formulation which is as simple as possible, and which can be sufficient for the problem at hand, thus considering a limited range of validity (Drucker, 1991). In many rock mechanics problems, an observational approach leading to empirical design formulations is probably appropriate. However, for more complex problems involving time scales which extend beyond usual engineering work (and the length of human life for that matter), the extrapolation of rock mechanics data becomes uncertain and risky (Ladanyi, 1982). This is when a thorough understanding of the inelastic behavior of the material becomes a prerequisite for answering engineering needs with a constitutive model.

The adoption of a linear viscous stress, or viscoelastic, (class I) model, (see review by: Fuenkajorn and Daemen, 1988) will give, for an unsupported underground opening excavated in an isotropic and homogeneous medium, a constant stress field that will develop instantaneously, simultaneously to the driving. The viscoelastic flow of the material around the excavation then proceeds without any modification to the stress field, either in time or in space. However, it is well known now that such models cannot be used to describe the actual rheological behavior of rocksalt under such loading conditions, even if a plastic component (or threshold) is introduced (e.g. Lindner, 1983; Fuenkajorn and Daemen, 1988).

On the other hand, models that consider nonlinear stress–strain relationships for the rock mass generate a stress redistribution around underground openings, even if it is assumed that the far field stresses remain constant. Such a stress redistribution is required to insure the continuity of the strain field. As the stress redistribution around an excavation proceeds, a part of the stress initially imposed on the highly stressed regions (usually near the excavation walls) is gradually transferred toward the regions that are initially less loaded (Edelstein and Hult, 1983), so that the stress state tends progressively toward a stationary condition. This stress redistribution induces transient inelastic behavior in the structure as the wall convergence rate of the excavation decreases progressively with time (Prij and Mengelers, 1981; Blais, 1988), as can be seen from curve (a) in Fig. 2. Because the general shape of the convergence curve shows a transient phase resembling that of laboratory creep tests, it has sometimes given rise to ambiguities regarding the distinction between the transient flow of the material, which is related to its hardening (as is the case in the primary creep phase in a laboratory creep test, for instance) and the transient behavior of the structure. During the stress redistribution period, the material flows not only according to the varying stress state, but also according to its inherent transient rheological behavior. The former cause depends on excavation geometry and on stress–strain non-linearity, while the latter stems from the hardening of the material itself.

Both aspects are related in a complex manner, as each change of stress produces new transient inelastic flow of the material, leading to further stress redistribution because the hardening of the material influences its stress–strain (and stress–strain rate for that matter) relationship. In modeling this phenomenon, the interaction between the two processes modifies the rate at which the stresses are redistributed around the excavation, thus producing a longer period of noticeable stress variation in the structure when compared to what is obtained by using classical creep laws (see Fernandez and Hendron, 1984, and the discussions of Morgan and Krieg, 1988, Senseny, 1990, and Aubertin et al., 1990). Appropriately enough, it has been frequently underlined by some authors that the stationary flow around underground openings has seldom been formally identified, even for old workings (of hundred of years) instrumented for long periods of time (Bérest, 1987; Borm and Haupt, 1988). In standard laboratory tests, however, isostatic loading conditions usually lead to steady-state flow of the material within a few days to a few months, depending on the imposed temperature, stress, or strain rate (Wawersik, 1985).

In order to predict the long term behavior of underground openings in rocksalt, many have adopted non-linear viscous stress (class I) models obeying a solely elastic/steady-state creep law, such as the power creep (or Norton) function (e.g. Preece, 1987; Mottahed and Ong, 1988). Such a model does in fact produce a transient flow of the structure (Fig. 2, curve b), related in this case to the stress redistribution only (Prij and Mengelers, 1981; Morgan et al., 1986; Blais, 1988). One can then observe a progressive migration of the peak deviatoric stress (σ_e) farther from the excavation walls as the inelastic flow proceeds (Wallner, 1984; Rolnik, 1988). Unfortunately, such relatively simple models have not given satisfactory results, under usually accepted predictive technology assumptions (as defined by Munson and Wawersik, 1991), when compared to the actual behavior of underground openings, as the difference between the predicted (using numerical techniques such as FEM) and observed wall displacements often exceeds 100–200% as shown by

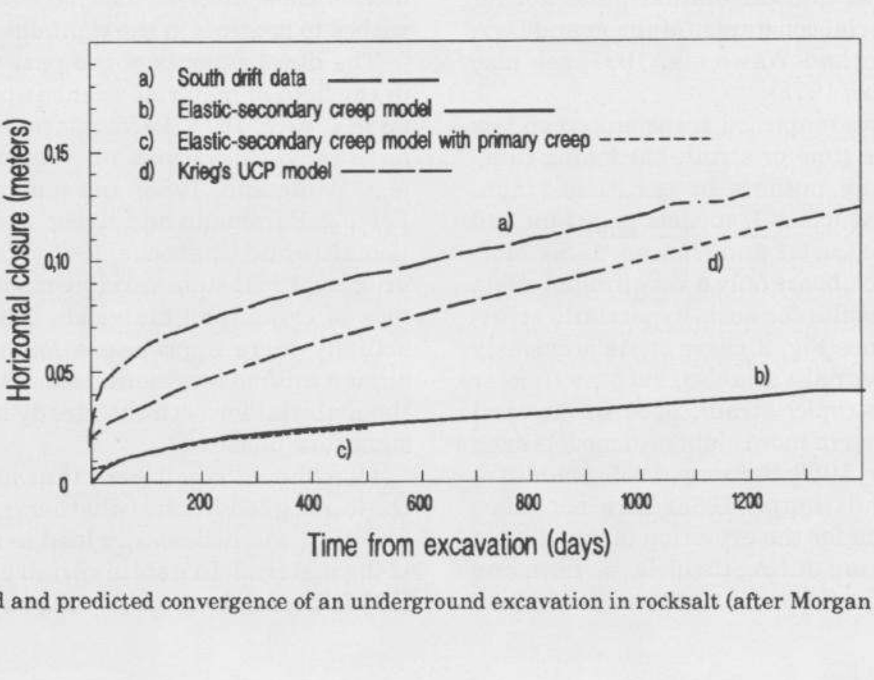

Fig. 2. Observed and predicted convergence of an underground excavation in rocksalt (after Morgan and Krieg, 1988).

recent modeling results (e.g. Fig. 2, curve a and b; see also: Munson et al., 1989a; Morgan and Wawersik, 1989; Bérest, 1991). This is not surprising, considering the fact that such models completely neglect the transient (hardening) inelastic behaviour of the material, and its interaction with the stress redistribution. This may constitute a major source of discrepancy in modeling results, that adds to the numerous other imprecisions inherent to such work (including material property variations and the numerical hypothesis adopted). The use of a steady-state creep law, although not representative of the reality, can, however, be considered as an interesting tool that can be applied for developing analytical solutions for idealized geometric structures and for stationary stress distribution, or for investigating through numerical modeling the general effects of various factors on the overall behavior of the rock mass around the underground openings (Boyle and Spence, 1983). Nevertheless, ignoring such an important aspect of the behavior of rocksalt while designing underground openings or monitoring existing underground excavations can lead to important misinterpretations.

It could be shown, however, that such an empirical elastic/secondary creep law can describe fairly well the wall convergence of specific underground excavations, under a given set of geometrical and boundary conditions, if the constants used (such as the elastic modulus E, or the power law exponent n) are adjusted to fit empirically *in situ* rather than laboratory data (e.g. Prij and Mengelers, 1981; Morgan and Krieg, 1988, 1990; Frayne and Mraz, 1991). This, of course, is not a method which strictly follows the usually accepted predictive technology procedure, where *in situ* measurements serve only for validating the model and calculations, and not for correcting the material constants (Munson and Devries, 1991; Munson and Wawersik, 1991; see also discussion by Lambe, 1973).

The addition of an empirical transient creep law such as the various time or strain-hardening functions, used by many authors in partitioned nonlinear viscous stress (class I) models (e.g. Lux and Heusermann, 1985; Carter and Hansen, 1983; Morgan and Krieg, 1988), bears only a very limited effect on the modeling results for such hyperstatic structures (see for instance Fig. 2, curve c). As previously mentioned, the external variables, such as time or the equivalent (isotropic) strain, used in classical formulations (or even in more elaborate models such as those of: Lindner, 1983; Senseny, 1985; Handin et al., 1986; Borm and Haupt, 1988) does not allow proper consideration for the evolution of the state of the material occurring during its inelastic flow, and the ensuing modification to its stress–strain (and stress–strain rate) relationship. Thus again, the complex interaction between the stress redistribution phenomenon and the hardening of the material is largely neglected. One might further add that the addition of the pseudo-instantaneous plastic strain (ε^p) in a partitioned approach has also a minor influence on modeling results of underground structures (Morgan et al., 1986), if the interaction between ε^p and ε^c is not properly considered.

For these reasons, a large portion of the discrepancy between the predicted and instrumented behavior of hyperstatic structures, such as underground excavations, is increasingly attributed to the inadequacy of the constitutive model used, when these are compared to the real (physical and phenomenological) rheological behavior of rocksalt (Morgan et al., 1986; Munson et al., 1989a; Côme, 1990). These unsuccessful attempts have lead a part of the scientific community to recognize the importance of including, in a more rigorous fashion, the transient phase of the inelastic behavior of rocksalt in the constitutive law, if one wants to model correctly the time-dependent behavior of underground excavations. This has, in turn, prompted many researchers to investigate more thoroughly the hardening of the material, and to try to develop constitutive models that are compatible with the physical processes that control the inelastic flow, thus leaving behind the purely empirical approaches usually considered in viscous stress models, that are solely based on phenomenological analysis of standard laboratory test results. As the stress path and the strain (and strain rate) history are quite different between laboratory and *in situ* conditions, the availability of new constitutive models closely related to the deformation mechanisms involved has become necessary if one wishes to progress in the right direction.

The developments of the past twenty years or so in the field of material science and applied mechanics (e.g. Rice, 1971, 1975; Asaro, 1983; Poirier, 1985) have provided various micromechanical (class III) (e.g. Cailletaud, 1988) and macromechanical (class IV) (e.g. Bammann and Krieg, 1987; Delobelle 1988; Lemaitre and Chaboche, 1988; Marchand and Moosbrugger, 1991) state variable models for the inelastic flow of crystalline materials. These latter models, actually more appropriate for engineering needs, allow a unified representation of the inelastic flow of the material for both the steady-state and the transient flow phases.

One should recall here that any modification to the loading conditions (whether σ, $\dot{\varepsilon}$ or T) in a laboratory test, will necessarily lead to transient behavior of the material. In a state variable model, such as the SUVIC model developed by the authors (see Table 1),

TABLE 1

State variable models developed for rocksalt

Authors	Comments
Krieg, 1982 (see also: Lindner, 1983; Devries, 1988)	– Model includes one kinematic state variable – Kinetic law is a product of linear and exponential functions – Growth law includes dynamic and static recovery
Krieg, 1989 (taken from Morgan, 1991, Personal communication)	– Same as the previous one, but with a normalizing scalar state variable for isotropic hardening – Encouraging improvement when compared to the original version (see: Morgan and Krieg, 1990)
Munson and Dawson, 1982 (see also: Senseny and Hansen, 1987; Devries, 1988)	– Based on phenomenological characteristics of the hardening and recovery (potential) functions for the creep of rocksalt – Kinetic law expressed as a function of the steady-state creep law, which is the sum of three constitutive laws (two power law terms and one hyperbolic sine law term) related to the three deformation mechanisms controlling the inelastic flow, as identified by Munson and Dawson – Different growth laws for loading (hardening phase) and unloading (recovery phase) conditions – The kinetic law induces a different stress dependence for transient and steady-state behavior – One scalar evolutionary state variable, expressed as a function of three varying parameters – Produces isotropic hardening – Model is "strain based", and relies on the existence of a total (saturation) transient strain (ε_t^*), which is a characteristic of constant stress experiments and not of the material itself; under other loading conditions, ε_t^* is dependent on the mechanical history, and should be formulated as such
Munson et al., 1989a, 1989b (see also: Munson and Devries, 1991; Morgan and Wawersik, 1991)	– Same as the previous one, but with a modification of the kinetic law aimed at a better description of the non-linearity of hardening and recovery – Uses a Tresca-type flow criterion
Stein and Wetjen, 1988	– Model includes one scalar state variable (associated with the dislocation density) acting as an isotropic internal stress – No kinematic hardening – Kinetic law formulation is a power law function
Aubertin, 1989 (see also: Aubertin et al., 1991a, b, d)	– SUVIC model includes three states variables (B_{ij}, R and K) and three evolution laws that produce mixed hardening – Kinetic law is a power law function applied to the normalized active stress – B_{ij} is a deviatoric internal stress tensor, responsible for kinematic hardening – R is a scalar internal stress, acting as a yield surface, and associated with isotropic hardening – K is a normalizing scalar variable which also contributes to isotropic hardening – Can reproduce the behavior of rocksalt under monotonous loading, reverse loading and for rotation of the principal stresses
Russel et al., 1990	– Based on the cross slip controlled dislocation motion – Total stress is the sum of two (or more) internal stresses, which serve as state variables – Growth laws, which do not include static recovery, contain a logarithmic term and a power term – Kinetic law and growth laws are coupled together in a complex manner, which may cause numerical difficulties for integration of mathematical equations – the possibility of mixed hardening exists, but has not been explicitly defined by the authors
Freed and Walker, 1991	– Viscoplastic model proposed for class M (metal-like behavior) materials – Model includes two state variables (a back stress B_{ij} and a yield strength Y), which induce mixed hardening – The kinetic law and the two evolution laws are expressed as hyperbolic sine functions – Kinematic state variable B_{ij} is coupled to the scalar Y in its growth law – Evolution laws of B_{ij} and Y include both dynamic and static recovery – In the domain of power-law creep, the model induces a stress dependence with an exponent $n = 3$, similarly to the 'natural' theory of creep for dislocation climb

memory effects are represented by the instantaneous value of the state variables, whose evolution laws are formulated using differential equations. This type of formulation is advantageous when compared to integral formulations, such as the simplified endochronic (class II) model developed by Borchert et al. (1984), which usually necessitate the knowledge and proper treatment of the entire mechanical history of the material. Various state variable models developed for rocksalt are listed in Table 1, together with a few brief comments on their main characteristics.

EVOLUTIONARY STATE VARIABLE MODELS FOR ROCKSALT

The preliminary modeling work that has been done on rocksalt with some recently developed differential state variable models are very encouraging (see: Devries, 1988; Morgan and Krieg, 1988, 1990; Morgan and Wawersik, 1989; Munson and Devries, 1991). However, because most of these models lack some of the main features of rocksalt's mechanical behavior, they may not be able to describe all the essential characteristics of its inelastic flow (see Table 1), as discussed in the following.

Offering many similarities with the model of Malinin and Khadjinski (1972), the model of Krieg (1982) constituted pioneering work for rocksalt, and paved the way to many subsequent developments. Unfortunately, because the original Krieg model included a single kinematic state variable only, it was not complete enough to offer a good description of the inelastic behavior of rocksalt. For example, it can be seen from Fig. 3a, that Krieg's model only represents with a relatively poor accuracy the transient creep phase of rocksalt in an incremental creep test. Nevertheless, such a simple state variable model has helped to improve significantly the accuracy of modeling results for underground excavations when compared to the previously presented viscous stress models (see Fig. 2, curve d). The extension of Krieg's model for mixed (isotropic and kinematic) hardening does offer promising possibilities in that respect, as shown by the recent work of Morgan and Krieg (1990). However, this version of the model, which does not include a viscoplastic yield criterion (a disadvantage in the authors' opinion; see discussion by Aubertin et al., 1991b), still has to be validated under laboratory controlled conditions, for various stress paths and strain histories, before it can be used with confidence for *in situ* conditions.

The Munson and Dawson (1982) model, also inspired by some of the constitutive modeling work done for other crystalline materials such as metals, offers an original and somewhat peculiar formulation. The kinetic law is the sum of three stationary creep laws, expressed solely as functions of the applied deviatoric stress (no state variable is considered for the steady-state flow of the material), multiplied by a factor F which is different for 'hardening' and 'recovery' conditions. The value of this factor F depends on the existence of a total transient strain (ε_t^*), again expressed as a function of the applied stress only, which is a characteristic of creep test results (and not of the material behavior itself). A single scalar variable (ζ) is used, whose value is then described according to three parameters, Δ, δ and ε_t^*, all expressed as functions of the total deviatoric stress state. It should be mentioned here that the kinetic law formulation of the Munson–Dawson model leads to a stress dependence that is different in the transient and steady-state phase of the inelastic flow, thus conflicting with the usually admitted principle previously stated. The Munson–Dawson model has been modified by introducing a more elaborate mathematical formulation for F, and by using a Tresca-type yield criterion, rather than the von Mises criterion, (Munson and Devries, 1991).

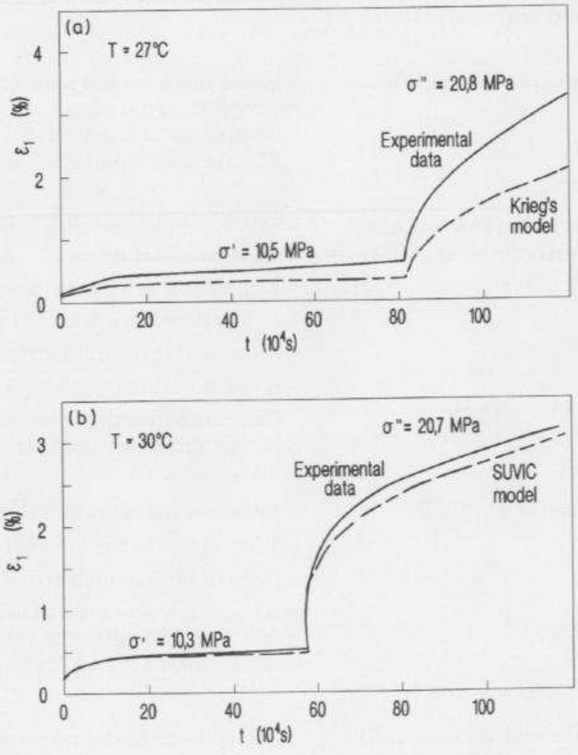

Fig. 3. Comparison between incremental creep test results, obtained from conventional triaxial compression testing, and predictions obtained with two unified models on Avery Island rocksalt. (a) Krieg's model (after Krieg, 1982). (b) SUVIC model (after Aubertin, 1989, with experimental data taken from Hansen and Mellegard, 1980).

According to Munson and Wawersik (1991), the Munson-Dawson model is "strain based" because the transient behavior is dependent on the accumulated strain. This, together with the isotropic hardening nature of the kinetic law, may limit the applicability of the model for complex loading conditions. Nevertheless, very good predictive results have been obtained with the modified Munson–Dawson model (Munson et al., 1989a, 1989b; Munson and Devries, 1991). Some of these good predictive results can, however, be puzzling, considering the intrinsic dispersion of the geomechanical data (and of the rocksalt properties), the hypothesis adopted to describe some of the 'missing' mechanical history of the material tested in the laboratory, and the simplifying assumptions made for the description of the mechanical behavior of rocksalt (see Table 1) and of the surrounding rocks (such as an elastic-purely plastic behavior with a Drucker–Prager yield criterion for instance; see Munson and Devries, 1991).

The Stein and Wetjen (1988) model, although based on generally sound physical concepts, and offering some similarities with other viscoplastic models, is too simple to be considered as a valid candidate for describing rocksalt behavior. The fact that only a scalar state variable is included, thus producing isotropic hardening due to expansion of a yield surface, should be enough to discard it.

The model by Russel et al. (1990) does appear to offer some promising capabilities. However, only a uniaxial formulation for monotonous loading conditions is available (to the authors knowledge) so that the exact contribution of each state variable (internal stress) to the mixed hardening of the material is still unknown. The model seems to be in its development phase, so that a final judgement is not possible at this stage. However, one can nevertheless notice that the coupling between the growth laws and the kinetic law might cause some numerical problems for the mathematical treatment of the differential equations. Also, the missing static recovery function in the evolution laws may limit the applicability of the model for complex mechanical history over long a period of time.

The Freed and Walker (1991) model follows closely the usual approach of differential state variable models developed for metals, extended here for class M (metal-like behavior) materials. It presents many similarities with the SUVIC model, especially in its physical basis. Accordingly, this model seems to offer many of the main features needed for rocksalt as defined by Aubertin et al. (1991a, 1991b). However, the numerical treatment of the model's equations may be cumbersome, considering the very strong non-linearity of the kinetic law and of the growth laws (expressed by using sinh functions), and the coupling between the evolution law of the two state variables. As is the case with the SUVIC model (Aubertin et al., 1991b, 1991c), the Freed and Walker (1991) model can be reduced to a power law expression with a stress exponent n having a value of 3, which is considered to be 'natural' for inelastic flow controlled by dislocation motion. This model has not yet been validated experimentally with rocksalt, but the few experimental results with a LiF-22% CaF_2 hypereutectic salt, presented by its authors, appear to be promising.

The SUVIC model (Aubertin et al., 1991a, 1991b, 1991d) based on the use of a viscoplastic yield criterion and of an associated normality flow rule, includes three evolutionary state variables which induce a mixed hardening of rocksalt. The model has been validated under various isostatic loading conditions as can be seen, for instance, in Figs. 1a, 1b and 3b. In this last case, one can notice that the accuracy of the representation of the transient creep phase in an incremental creep test is much better than that of Krieg's original model.

Finally, one should mention that other state variable models that have been developed for other crystalline materials, such as metals and ice, have also been investigated by the authors and have been found to be inappropriate for describing the inelastic behavior of rocksalt (Aubertin, 1989).

REMAINING QUESTIONS

Before proposing a definitive constitutive model for predicting the stress, strain and displacement fields around underground openings, there are still a few important questions that remain to be clarified regarding the inelastic behavior of rocksalt. Among these, one can mention:

(i) the exact nature of the viscoplastic yield criterion (von Mises, Tresca, or other),

(ii) the possibility of having to include rotation and distortion of the yield surface,

(iii) the exact nature of the mixed hardening, that is, the relative contribution of isotropic and kinematic hardening of the material, as well as the best experimental procedure to define it.

Furthermore, it must be stated here that the application of predictive technology to underground excavations should be considered with great care for model validation. In the authors' opinion, the validation of a constitutive model should first be thoroughly considered by using well-controlled laboratory test results, starting with isostatic loading conditions (such as CTC test) with increasing complex mechanical history (such as stress dip, stress rotation, strain rate increase, etc), and continuing with instrumented hyperstatic structures such as thick walled cylinder experiments (see Morgan and Wawersik, 1991). Only then will confidence be gained towards *in situ* predictive technology, where the apparent accuracy of the results can be misleading, as shown by numerous examples in the literature on rocksalt.

The major points to be considered are:

(a) the inherent variability of the mechanical properties of rocksalt and, consequently, of the various parameters included in constitutive models,

(b) the imprecise knowledge of the pre-excavation stress field,

(c) the possible occurrence of cracks during the inelastic flow outside of the ductile regime.

The last item should be a major concern because many observations have shown that deep underground excavations in rocksalt induce fracture near the opening walls, where the initial deviatoric stress is high and the mean stress is low. Very few existing models allow the consideration of this aspect of the inelastic behavior of rocksalt (e.g. Desai and Zhang, 1987; Cristescu, 1991; Aubertin et al., 1993). Accordingly, this should constitute one the focal points of future research activities.

CONCLUSIONS

The problems associated with the selection of an appropriate constitutive law for the prediction of the stress, strain and displacement fields around underground openings excavated in rocksalt (or potash) has been presented and discussed. It has been shown that a model allowing the proper consideration of transient inelastic flow is needed.

The inelastic behavior of crystalline materials, such as rocksalt and potash, is inherently complex and very sensitive to its mechanical history, so that empirical models based on curve fitting expressions of experimental results obtained under simple loading conditions are usually inadequate. Recent developments have shown that a valid constitutive model has to be based on sound physical basis, so that some confidence may be placed on the laboratory test results which have to be extrapolated to *in situ* conditions over long periods of time.

It is probably not reasonable to expect a constitutive model to reproduce all the different characteristics of the mechanical behavior of the material. The objective of constitutive modeling is to obtain a set of mathematical equations as simple as possible, and which are sufficient to describe the material behavior for the problem at hand, within the range of approximate validity.

For the design of underground excavations, the interaction between the stress redistribution within the structure and the transient (hardening) inelastic behavior of rocksalt requires that a state variable model be considered. Such a model should be able to describe the main characteristics of the mechanical behavior, including static and dynamic recovery of the microstructure, and the mixed hardening nature of rocksalt. Among the various state variable models available, and briefly reviewed in this article, the SUVIC model appears to be, for the time being, the most complete tool available.

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