

## A Damage Model for Rocksalt: Application to Tertiary Creep

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### ABSTRACT

Recent progress in material sciences and applied mechanics has allowed a better understanding of the physical processes governing the inelastic behavior of various materials, including rocksalt. This has prompted the development of state variable models, which can take into account microstructural evolution during inelastic flow, thus enabling hereditary effects to be considered properly. Such a model has recently been proposed by the authors for the three-dimensional viscoplastic behavior of rocksalt in the ductile regime. In this paper, a simplified version of the model, developed for steady-state creep under triaxial compression testing conditions, is extended in order to describe tertiary creep where cataclastic processes play a major role in the inelastic flow. In this extended model, named SUVIC-D, the kinetic law governing the accelerated creep strain rate is formulated using the continuum damage theory, in which the progressive deterioration of the material properties is related to the evolution of a damage variable, as expressed through a growth law.

After describing the mechanical behavior of rocksalt in the ductile, brittle and semi-brittle regimes, the model equations are presented and assessed by using creep tests results on rocksalt. Some of the characteristics of the model are then discussed in relation to other models used for tertiary creep of rocks.

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### INTRODUCTION

It is customary to assume that the inelastic behavior of rocksalt (and potash) is controlled by ductile (fully plastic) deformation mechanisms, involving various dislocation processes. Consequently, the rate-dependent behavior of these soft rocks is commonly treated independently of the mean stress value. It is known however that the inelastic behavior of rocksalt, like that of most other rocks (Kirby, 1983; Cristescu, 1989; Ladanyi and Aubertin, 1990), can also include a contribution from fracture, owing to crack and void initiation and propagation. Numerous experimental results have shown that many mechanical characteristics of rocksalt, such as the flow stress and failure strength, are in fact a complex function of the loading conditions, including strain rate, temperature and mean stress (Wawersik and Hannum, 1980; Cristescu and Hunsche, 1991). However, because constitutive models used with rocksalt have been mainly developed for isovolumetric processes, independent of the mean stress, there is still a relative scarcity of constitutive laws and of experimental data applicable to its inelastic behavior when cataclastic processes are active, that is, in the so-called brittle and semi-brittle regimes.

In this paper, a unified viscoplastic model with evolutionary state variables, named SUVIC, is extended in order to describe the secondary and tertiary creep of rocksalt, up to failure. After a brief presentation on the mechanical behavior of rocksalt and on the physical processes leading to the gradual transition from a ductile regime to a (semi-brittle and) brittle regime, the authors present the model equations, which make use of the continuum damage mechanics (CDM) theory. As a particular application of the proposed model, the kinetic law and the growth law of the damage variable are used to describe the tertiary creep of rocksalt from previously published results. Some of the model characteristics are finally discussed in relation to experimental results and to other models developed for describing tertiary creep of rocks.

### THE BEHAVIOR OF ROCKSALT AND THE BRITTLE-DUCTILE TRANSITION

The response of rocksalt to external loads is greatly influenced by the rate of loading and by the time under load. The effect of time, or of the strain rate, on the mechanical behavior of rocksalt under engineering-relevant conditions, has essentially two

main origins: (i) motion of dislocations in the crystalline structure and (ii) micro-fracturing. The relative contribution of these two broad classes of deformation mechanisms depends on many factors, among which predominate the temperature and the stress state, including the magnitude of both the deviatoric stress (relative to the yield and failure strengths) and the mean normal stress.

The question of the gradual transition from brittle (cataclastic) to ductile (fully plastic) flow of rocks has been the subject of numerous studies in the last two decades or so (e.g. Paterson, 1978; Kirby, 1983; Heard, 1985; Ross and Lewis, 1989; Evans et al., 1990; Bernabe and Brace, 1990; Murrell, 1990). Following these, it is now customary to identify three regimes of inelastic flow in crystalline rocks, according to the deformation and failure modes (Kirby and McCormick, 1984; Carter and Tsenn, 1987; Ladanyi and Aubertin, 1990). These can be identified as the brittle, semi-brittle and ductile regimes. Each are briefly discussed in the following, in relation to conventional triaxial compression (CTC) tests under constant strain rate (CSR) and constant stress (creep) conditions, which can both be used to study the inelastic behavior of rocks.

Figures 1 and 2 illustrate schematically, for rock-salt, the stress conditions under which these various inelastic flow regimes predominate. The first of these two figures is for CSR test conditions; the stress conditions are expressed in  $(\sigma_1 - \sigma_3)$  vs.  $\sigma_3$  space, with the corresponding stress-strain curves. Figure 2 is for creep test conditions. The stress conditions in Fig. 2a are expressed in strain-time space for constant confining pressure. The onset strain  $\epsilon_0$  of the tertiary creep phase is considered to be approximately constant for a given confining stress, whatever the axial stress (Cruden, 1974). The influence of the confining stress on the former is illustrated in Fig. 2b.

### The ductile regime

Rocksalt behavior in the ductile regime is probably the best known of the above three. It is dominated by intracrystalline dislocation motion which induces fully plastic (essentially isovolumetric) deformation. Nevertheless, some cracks may appear in polycrystalline aggregates in order to accommodate small local displacements, mostly near grain boundaries where stresses are higher (Hansen, 1985; Aubertin, 1989). Among the deformation mechanisms involved, dislocation glide, climb and cross slip are active, with the latter being considered dominant for the usual loading conditions (Aubertin et al., 1991a, 1991b).

Under CTC laboratory testing, the ductile regime

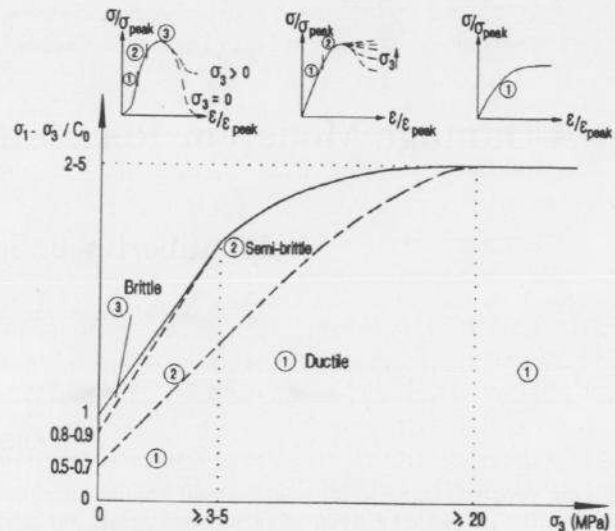


Fig. 1. Schematic representation of the inelastic behavior of rock salt in conventional triaxial compression tests at a constant strain rate. The peak strength (solid line) and boundaries between the three regimes (dashed lines) vary according to the strain rate and temperature.

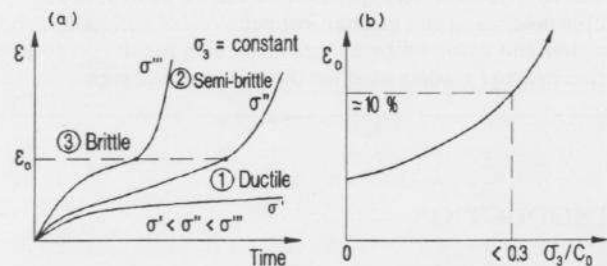


Fig. 2. (a) Typical creep curves at constant confining stress in conventional triaxial compression tests (after Cruden, 1974). (b) Effect of the confining pressure on the onset strain of tertiary creep.

can be observed for rock salt at or above room temperature, for a strain rate below  $10^{-3}$  to  $10^{-4} \text{ s}^{-1}$ . As is the case with other rocks, ductility is enhanced by an increase in confining pressure; fully plastic mechanisms are usually considered dominant if the minor principal stress  $\sigma_3$  exceeds about 20 MPa. It can also be predominant at much lower values of  $\sigma_3$ , if the strain rate is very low and/or if the deviatoric stress state is below the crack initiation threshold, which is about 50–70% of the failure strength in uniaxial compression ( $C_0$ ) for instance. The ductile regime is represented by regime 1 in Figs. 1 and 2.

In the ductile regime, the inelastic behavior of rock salt is very similar to that of many metals so that the same type of constitutive models have been used (Munson and Wawersik, 1991), including unified models with state variables such as the SUVIC model proposed by the authors (Aubertin et al.,

1991a, 1992). Because the microscopic yield strength (or critical resolved shear strength on active dislocation slip systems) of rocksalt is very low in a virgin (or annealed) state (Barber, 1985; Haasen, 1985; Aubertin, 1989), the inelastic flow of the material occurs almost from the start of the deviatoric loading. Rate effects are here very pronounced, and failure in compression (if it exists) must occur at a very large strain due to crack initiation at dislocation pile-up (e.g. Ghandi and Ashby, 1979; Evans and Hsueh, 1981; Hayhurst and Leckie, 1990).

Needless to say, the behavior of rocksalt in the ductile regime is almost completely independent of the confining pressure (or of the mean normal stress). However, the frontier between this regime and the semi-brittle regime (that is, the crack initiation threshold) strongly depends on it. This boundary is also dependent on strain rate and on temperature, the ductile regime being favored by a lower strain rate and by an increased temperature.

### The brittle regime

At the other end of the spectrum of rocksalt inelastic behavior lies the brittle regime (regime 3 in Figs. 1 and 2). This regime is said to occur when cataclastic processes, such as crack initiation (and propagation) and frictional sliding, play the controlling role in the rheological behavior of rocks. For rocksalt, it is dominant under most tensile stress conditions, and also under compressive stress at relatively low temperature ( $T \leq 0.25$  to  $0.30 T_m$ , where  $T_m$  is the absolute melting temperature), at low minor principal stress ( $\sigma_3 \leq 3$  to  $5$  MPa), for a strain rate higher than about  $10^{-6}$  to  $10^{-7} \text{ s}^{-1}$  and for a deviatoric stress  $\sigma$  which approaches the failure strength  $\sigma_f$  ( $\sigma \geq 0.8$  to  $0.9 \sigma_f$ ). Various distinct physical mechanisms can contribute to cataclastic flow in the brittle regime (and also in the semi-brittle regime), including cleavage by crack propagation and void creation by dislocation pile-up at grain boundaries (e.g. Carter and Kirby, 1978; Paterson, 1978; Ghandi and Ashby, 1979; Evans and Hsueh, 1981; Atkinson, 1987; Nix, 1988).

In the brittle regime, the propagation of fractures often leads to their coalescence and to the eventual formation of localized failure planes. Because of the frictional nature of the deformation mechanisms in the brittle regime, the flow stress and failure strength of rocksalt are highly dependent on the confining stress. The failure strength of rocksalt in the brittle regime appears as a linear function of the confining pressure (or of the mean stress), thus obeying a Coulomb-type failure criterion. Accordingly, as is the case with other rocks (Paterson, 1978; Carter and Tsenn, 1987), the inelastic flow is accompanied by dilation.

In the brittle regime, the relative contribution of dislocation motion to inelastic flow can be considered almost negligible and consequently, the flow stress and failure strength of rocksalt are only weakly influenced by strain rate and temperature when compared to the ductile regime. In fact, when the loading conditions are very close to the failure strength of rocksalt in the brittle regime, the velocity of crack propagation becomes close to critical, thus leading to an almost dynamic rupture of the material.

When creep tests are taken near to failure, the creep curves are found to be S-shaped, with no or a very short period of stationary creep, followed by a clearly distinguishable accelerating tertiary creep phase. It is often believed that the transition to tertiary creep in rocks occurs only above some critical stress (or strain) level, which would be a function of the experimental conditions, especially of  $T$  and  $\sigma_3$  (Cruden, 1974; Kranz and Scholz, 1977); such an assumption should also be true for rocksalt in the brittle (and semi-brittle) regime (see Fig. 2).

The stage of development of constitutive equations for cataclastic (brittle) deformation of rocksalt is less advanced than that for ductile deformation (Senseny and Hansen, 1987). There is also a relative scarcity of data available for cataclastic deformation. Because rate-effects are less pronounced, many models used to describe the inelastic behavior of rocksalt near or at failure, in the brittle regime (and also in a part of the semi-brittle regime), neglect time-effects. Thus, similarly to the traditional approach in rock mechanics, it is often treated by using empirical rate-independent failure criteria (Hunsche, 1990; Langer, 1991).

Rate-independent and more appropriate rate-dependent models have also been proposed for rocksalt in the brittle (and semi-brittle) regime based on plasticity theory with an associative or a non-associative flow rule, using hardening and/or softening functions (e.g. Butcher, 1980; Desai and Varadarajan, 1987; Desai and Zhang, 1987; Cristescu and Hunsche, 1991), following an increasingly popular approach in rock mechanics (e.g. Cristescu, 1989; Zienkiewicz et al., 1989).

More fundamental models have also been inspired by recent developments in fracture mechanics of rock (e.g. Atkinson, 1987; Dayre, 1987; Meredith, 1990; Charlez, 1991), many of which using a variation of the model by Brace and Bombolakis (1963) on wing crack propagation with frictional sliding (e.g. Costin, 1983; Kemeny and Cook, 1987; Hallam and Ashby, 1990; Kemeny, 1991). Pursuing this direction, one is led to consider state variable models by the use of the Continuum Damage Mechanics (CDM) theory, developed originally for metals (Kachanov,

1986; Lemaitre and Chaboche, 1988), and extended since for various materials including concrete and rocks (Butcher, 1983; Costin, 1983; 1987; Krajcinovic and Lemaitre, 1987; Yazdani and Schreyer, 1988; Singh and Digby, 1989; Ofoegbu and Curran, 1991; Charlez, 1991). This is the chosen path for the extension of the SUVIC model to the brittle and semi-brittle regimes in the present paper.

### The semi-brittle regime

Between the regimes of ductile and brittle inelastic flow of rocksalt, there exists a relatively large transition domain, often called the semi-brittle regime (regime 2 in Figs. 1 and 2). Here, dislocation motion acts simultaneously with cracks, with the two processes interacting (Carter and Kirby, 1978; Carter and Tsenn, 1987; Ross and Lewis, 1989; Evans et al., 1990).

The passage from the ductile to the semi-brittle regime is initiated when the deviatoric stress state exceeds the crack initiation threshold, which is a function of the confining stress, temperature and strain-rate. The debut of the semi-brittle regime may be taken as the onset of dilatancy, which for rocksalt seems to appear at a deviatoric stress exceeding about 50–70% of the failure strength at low confining pressures (e.g. experimental results of: Wawersik and Hannum, 1980; Desai and Varadarajan, 1987; Spiers et al., 1989; Hunsche, 1990). Macroscopically, the identification of fully plastic (ductile) flow is usually based on the lack of pressure sensitivity of the inelastic behavior and also, occasionally, on the examination of microstructures of deformed samples (Evans et al., 1990).

On the other hand, the suppression of crack extension by the minor principal stress, which induces stable and finite microfracture propagation, is a gradual process that favors the change from a brittle to a semi-brittle behavior.

In the semi-brittle regime, the strains are no longer localized on or near faults or fracture planes, but are well distributed throughout the entire volume of the material, on a shorter average length scale. The operative microfracturing produces some dilation, but at a rate which becomes progressively smaller than that observed in the brittle regime. Thus, an increase of the minor principal stress in the semi-brittle regime of rocksalt increases the flow stress and the failure strain and strength.

An increase in the confining pressure also increases the crack initiation threshold. An upper bound to the threshold between the ductile and semi-brittle regimes of rocks is sometimes given empirically as a linear criterion of the type  $\sigma_1 - \sigma_3 = x\sigma_3$ , where  $x$  is a constant whose value is close to unity

(Evans et al., 1990). However, for rocksalt, which is probably one of the most ductile rocks, this relationship largely over-estimates this boundary (i.e. over-estimates the threshold stress  $\sigma_1 - \sigma_3$ ). Furthermore, it is not clear if the linear nature of this criterion is maintained (as proposed by Spiers et al., 1989, and Hallam and Ashby, 1990) or if a Griffith-type non-linear criterion would be more appropriate. Further work is needed on this aspect.

The axial stress vs. strain relationship within the semi-brittle regime usually indicates strain-hardening. It should be noted, however, that despite producing a similar effect on the stress-strain curve, namely an increasing flow stress with a progressively decreasing hardening modulus  $h$  (where  $h = d\sigma/d\varepsilon$ ), ductile and cataclastic deformation mechanisms produce quite different effects within the material itself. In the former case, the increased interaction of dislocations with obstacles during inelastic flow produces hardening of the material, which increases the yield strength. Cataclastic mechanisms, on the other hand, rather induce a gradual degradation of the mechanical properties, which appear in the stress-strain curve as a progressive reduction of the deformation resistance of the material, with little or no increase in the yield strength.

Many of the constitutive models used for rocksalt in the semi-brittle regime are inspired by classical empirical creep laws. However, these are neglecting some important characteristics of the material behavior. Restrictions applicable to these types of model for the ductile behavior of rocksalt (Senseny and Hansen, 1987; Aubertin et al., 1993), such as the inadequate treatment of memory (or hereditary) effects, are also relevant to the semi-brittle behavior of rocksalt. Because these purely empirical models are often based on invalid external variables, such as time or isotropic strain, lacking the proper physical insight into the rate-dependent behavior of rocksalt, they are probably not appropriate for many engineering needs.

Furthermore, treatment of the two types of active processes is usually not properly coupled in constitutive modeling of rocksalt (Senseny and Hansen, 1987). In the authors' opinion, this might not be an acceptable treatment of the inelastic behavior of rocksalt. That is why the proposed model will consider an intrinsic coupling between cataclastic and fully plastic processes.

### A DAMAGE MODEL FOR DESCRIBING TERTIARY CREEP OF ROCKSALT

As previously mentioned, many models have been proposed to describe the inelastic behavior of rock-

salt in the ductile regime, using various approaches such as non-linear viscous stress partitioning models and evolutionary state variable models (Aubertin et al., 1993). Among the latter, the SUVIC model is a unified viscoplastic model with three state variables which has been developed by the authors (Aubertin et al., 1991a).

This model is extended here by using the CDM theory, and applied to accelerating creep tests results on rocksalt in the semi-brittle and brittle regimes. In order to treat, in a coupled manner, the two types of deformation mechanisms involved, it was necessary to produce a model formulation that allows the kinetic law to be expressed both as function of the hardening of the material, due to fully plastic processes, and of damage due to cataclastic deformation.

For CTC tests, the extended model, named SUVIC-D, is formulated as follows for the total axial strain rate:

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^i \quad (1)$$

$$\dot{\epsilon}^e = \dot{\tilde{\sigma}} / E \quad (2)$$

$$\dot{\epsilon}^i = A \left\langle \frac{|\tilde{\sigma} - B| - R}{K} \right\rangle^N \text{sgn}(\tilde{\sigma} - B) \quad (3)$$

$$\tilde{\sigma} = \frac{\sigma}{1 - D} \quad (4)$$

$$\sigma = \sigma_1 - \sigma_3 \quad (5)$$

where  $\tilde{\sigma}$  is the net stress:  $\tilde{\sigma} = \sigma$  for undamaged material ( $D = 0$ );  $\tilde{\sigma} > \sigma$  for damaged material ( $0 < D \leq D_c$ )

In these equations,  $\dot{\epsilon}^e$  and  $\dot{\epsilon}^i$  are the elastic and inelastic strain rates respectively;  $E$  is the elastic modulus;  $\text{sgn}$  is the signum function ( $\text{sgn}(x) = 1$  for  $x \geq 0$ ,  $\text{sgn}(x) = -1$  for  $x < 0$ );  $B$ ,  $R$  and  $K$  are the three evolutionary state variables included in the SUVIC model, and whose evolution laws have been described elsewhere (Aubertin et al., 1991a);  $D$  is a damage variable whose value controls the net stress acting on the 'undamaged' portion of the material;  $D$  equals 0 in the undamaged state, and grows until it reaches  $D_c$  (where  $D_c \leq 1$ ) at failure.

The evolution law for variable  $D$  follows the general formulation of the CDM theory (Lemaitre and Chaboche, 1978):

$$\dot{D} = \dot{D}[\sigma, D, T, Y] \quad (6)$$

Equation 6 is expressed as a function of the overall stress state (including the deviatoric and mean

stresses), the accumulated damage  $D$ , the temperature and the value of all the state variables  $Y$ . Assuming a negligible effect of  $Y$  on  $D$ , as is usually the case (Lemaitre and Chaboche, 1988), one can obtain, for CTC tests at constant temperature, the following expression often used for tertiary creep of metals (Chan, 1988; Hayhurst and Leckie, 1990):

$$\dot{D} = f_1[D] f_2[\sigma] \quad (7)$$

where  $f_1$  is a function of the damage variable  $D$  and  $f_2$  is a function in which the stress state is expressed from the value of the principal stresses ( $\sigma_1$  and  $\sigma_3$  for CTC tests). Various specific formulations have been used in order to define functions  $f_1$  and  $f_2$  (Lemaitre and Chaboche, 1978, 1988; Chan, 1988). Among these, the Kachanov-Rabotnov formulation proposed for tertiary creep of metals (under uniaxial loading) is of particular interest for rocksalt. In a slightly modified form, it can be written as follows:

$$\dot{D} = \dot{D}_0 (1 - D/D_c)^{-k} \sigma^r \quad (8)$$

where  $\dot{D}_0$ ,  $D_c$ ,  $k$  and  $r$  are considered material constants. This damage function is used in SUVIC-D for describing tertiary creep.

## MODEL ASSESSMENT FOR TERTIARY CREEP

In order to demonstrate that the SUVIC-D model can describe properly the tertiary creep of rocksalt, two creep test results leading to failure have been taken from the literature and analyzed according to the model equations. These creep curves are shown in Fig. 3.

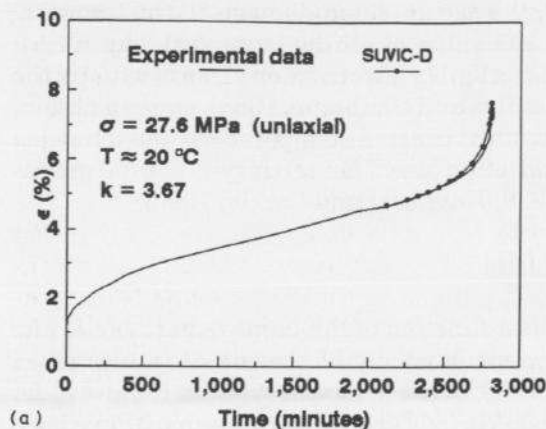
To obtain a simple analytical solution, the following simplifying assumptions are adopted here for tertiary creep of rocksalt under CTC loading:

(i) The variation in the elastic strain rate due to damage is neglected because it is considered to be small compared to the inelastic strain rate.

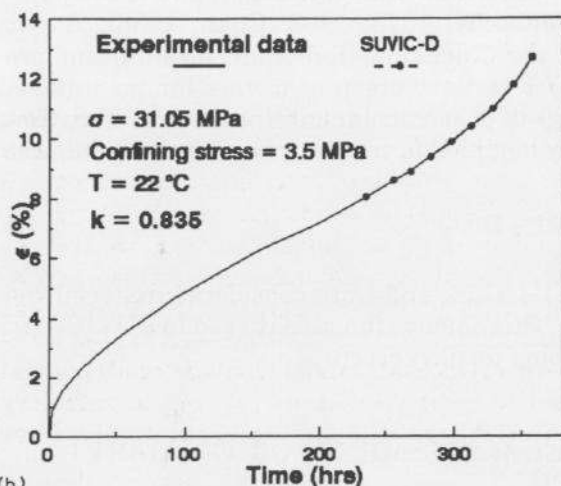
(ii) The values of exponent  $N$  and of parameter  $K$  in the kinetic law of the SUVIC model are fixed at 3.7 and 12.7 MPa respectively, which seem to be the nominal values appropriate for various rocksalts (Aubertin, 1989).

(iii) The value of the active stress  $\sigma_a$  (where  $\sigma_a = |\sigma - B| - R$ ) is considered as a constant during the steady state creep of rocksalt; it equals about 30% of the applied stress (Aubertin et al., 1991b).

(iv)  $D$  is considered to be a scalar, which is an acceptable assumption for monotonous CTC tests.



(a)



(b)

Fig. 3. Experimental creep (axial strain vs. time) curves for rocksalt and curves obtained from SUVIC-D. (a) Experimental data taken from Potts (1964). (b) Experimental data taken from Wawersik and Hannum (1979).

For a creep test, the growth law expressed by equation (8) can now be integrated to give:

$$\frac{D}{D_c} = 1 - \left[ 1 - \frac{t}{t_c} \right]^{1/(k+1)} \quad (9)$$

where  $t_c$  is the time to failure corresponding to  $D_c$ ;  $t = 0$  at the beginning of the tertiary creep phase.

In equation (9),  $t_c$  is a function of the stress state, which does not need to be defined explicitly for this model assessment.

In the case of the creep test result illustrated in Fig. 3a, the value of the constant  $A$  (equation 3) in the undamage condition ( $D = 0$ ) is first determined from the steady-state portion of the creep curve. The evolution of  $D$  is then calculated as a function of time by relating  $D$  to the increase in the inelastic strain rate obtained from the tertiary creep portion of the creep curve. This evolution is presented in Fig. 4a. To obtain the value of the constant  $k$ , the value of log

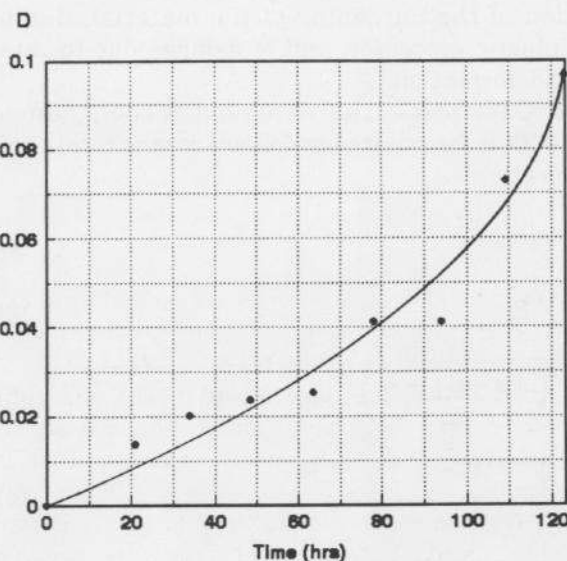
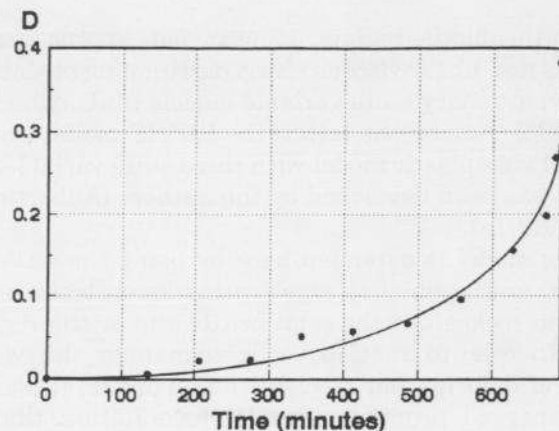


Fig. 4. Calculated values of the damage variable  $D$  expressed as a function of time during tertiary creep. (a) Data calculated from the experimental results of Potts (1964) (see Fig. 3a). (b) Data calculated from the experimental results of Wawersik and Hannum (1979) (see Fig. 3b).

$(1 - D/D_c)$  is plotted against  $\log(1 - t/t_c)$ , as shown in Fig. 5; according to equation (9), the slope of the straight line approximation so obtained is equal to  $1/(k+1)$ . At this point, all the numerical values of the parameters required to model the tertiary creep phase curve are known. Substituting them into equations (3) and (9), one obtains the filled circles shown in Fig. 3a.

Because the creep curve showing a tertiary creep phase in Fig. 3b does not seem to possess a steady-state phase, the value of the constant  $A$  for that rocksalt has been determined from the minimum creep rate. Figure 4b reproduces the evolution of the damage variable  $D$  as tertiary creep proceeds;  $k$  is obtained as in the previous case. The filled circles in Fig. 3b represent the proposed model description of the experimental creep data.

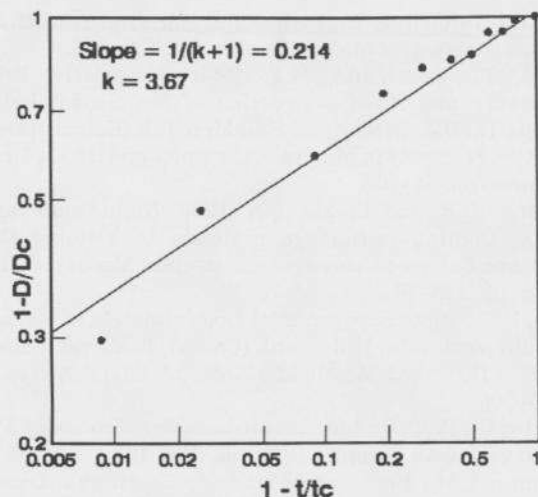


Fig. 5. Example of the graphical determination procedure for the value of the constant  $k$  in SUVIC-D for a creep test leading to failure (from the experimental data of Potts, 1964; see Fig. 3a).

## DISCUSSION

Various theories have been used in order to describe the tertiary creep (or static fatigue) of rocks. As is the case with metals (Odquist, 1974), most of these studies have mainly focused on the relationship between (1) the time to failure and the stress state (Rzhevsky and Novik, 1971; Singh et al., 1971; Carter and Kirby, 1978; Das and Scholz, 1981) or the loading rate (Singh et al., 1971; Lajtai et al., 1991), and (2) between the strain at the onset of tertiary creep and both the time and the stress state (Cruden, 1974).

However, there has not been a great deal of interest in the description of the evolution of strain during tertiary creep of rocks. Nevertheless, a few models have been proposed, often formulated in terms of internal variables, reflecting relative crack dimensions as a function of time and of the imposed stress state. Among these, the recent model of Kemeny (1991) is of particular interest. This model can be considered as a CDM model where the damage variable is expressed as a function of the ratio between the length and separation of sliding cracks ( $l/b$ ). Interestingly, the evolution law of the variable  $D$  in SUVIC-D (equation 8) can be shown to have a somewhat similar mathematical formulation to the growth law of the aforementioned variable  $l/b$  in Kemeny's model (see equation 15 in his paper). Accordingly, the  $D$  vs.  $t$  relationships presented in Fig. 4 resemble that obtained from Kemeny's model.

It should be noted that the damage variable  $D$  included in SUVIC-D is presented as a scalar, which is a correct assumption for monotonous loading under CTC conditions. However, because the physi-

cal nature of damage within polycrystalline materials is somewhat anisotropic (Costin, 1987; Lemaitre and Chaboche, 1988; Murakami, 1990), it is likely that the general three-dimensional formulation of the model will necessitate a treatment involving a vector or a second (or higher) order tensor description of the damage variable.

Furthermore, because only two (constant stress) creep tests on two different rocksalts were considered in this paper, the stress function  $f_2$  in equation (7) could not be formulated explicitly. Again however, generalization of the model will imply a specific mathematical formulation which must be adequate for describing the damage evolution under more general loading conditions.

## CONCLUSIONS

A unified viscoplastic model with state variables, applicable to steady-state creep test results, has been extended in order to describe the tertiary creep of rocksalt. The extended version of the model, named SUVIC-D, is formulated using a scalar damage variable whose evolution law follows a Kachanov-Rabotnov type formulation. It is shown that the proposed model can be used to describe the tertiary creep of rocksalt under conventional triaxial compression testing conditions in a very satisfactory way, even though simplifying assumptions have been postulated.

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